M Northwestern Medicine

Feinberg School of Medicine

A gentle introduction to meta-analysis

Applied Statistics Seminar

April 19, 2022

Jacob M. Schauer, Department of Preventive Medicine– Biostatistics, Northwestern University

What is this talk about?

Meta-analysis and statistics, right?

- What are effect sizes, and why should we care?
- Why would we combine effect sizes and how?
- What are some things to consider when combining effect sizes?
- Complexifying issues in analysis
- Some examples with the R package metafor.
 - Also weightr for one example
 - Feel free to code along while I talk!

What are the statistics of meta-analysis?

- We have already identified relevant studies and data within studies.
 - Dr. Muhammad's *Statistically Speaking* talk.



M Northwestern Medicine[®] Feinberg School of Medicine



9 studies examining the impact of antihistamines on runny nose severity for the common cold

- Outcome: Change in runny nose severity after 2 days
 - 4 different scales (0-3, 0-4, 0-8, 0-10)
- 2 different drugs: chlorpheniramine and doxylamine
- Some studies find statistically significant effects, some don't
- One study finds a negative effect

How do we make sense of all of these?

Morthwestern Medicine

Feinberg School of Medicine

Effect Sizes

What are they? Can we combine them?

What is an effect size?

Some statistical considerations for a single study

- Estimand θ
 - "True" effect
 - Parameter
- Estimates T
 - Function of the data
- Variance σ^2
 - Standard error: σ
 - Sampling or estimation error variance that decreases with sample size
- Confidence/credible intervals



What is an effect size?

Some statistical considerations for multiple studies

For *i* = 1, ..., k

- Estimands θ_i
 - "True" effects
 - Parameters
- Estimates T_i
 - Functions of the data
- Variances σ_i^2
 - Standard errors: σ_i
 - Sampling or estimation error variance that decrease with sample size
- Confidence/credible intervals



What we talk about when we talk about effect sizes

Some statistical considerations for multiple studies

- Estimands θ_i (effect size parameter)
- Estimates *T_i* (*effect size estimate*)
- The scale of estimands and estimates (effect size index)
 - Consider a two-armed study (Treatment vs. Control)
 - T-C mean difference: $\mu_T \mu_C$
 - T-C standardized mean difference (Cohen's *d* is "scale-free"...kind of): $(\mu_T \mu_C)/\varsigma$
 - T-C odds ratio (log transform), risk difference, ...
 - Correlation coefficient (arctan transform)

Where do effect sizes come from?

- To run a meta-analysis we need both the effect estimate T_i and variance σ_i^2 .
- To compute an effect size estimates and variances, we need data:
 - Raw data (unlikely for every or even most studies)
 - Summary statistics
 - Often reported in primary research

Effect size calculation is not always trivial

Example: Cohen's d in a 2-armed RCT

- $Y_{iT} \sim N(\mu_T, \varsigma^2)$ and $Y_{iC} \sim N(\mu_C, \varsigma^2)$ - $i = 1, ..., n_T$ and $i = 1, ..., n_C$
 - $-n=n_T+n_C$
- Cohen's $d = (\mu_T \mu_C)/\varsigma$
- Estimate (Glass, 1976)

• Bias correction (Hedges' g):
$$g = \frac{\Gamma(n^{-2}/2)}{\sqrt{\frac{n-2}{2}}\Gamma(n^{-3}/2)} \frac{\bar{Y}_T - \bar{Y}_C}{\sqrt{\frac{(n_T - 1)s_T^2 + (n_C - 1)s_C^2}{n-2}}}$$

• Approximate bias correction: $g \approx \frac{4n - 12}{4n - 9} \frac{\bar{Y}_T - \bar{Y}_C}{\sqrt{\frac{(n_T - 1)s_T^2 + (n_C - 1)s_C^2}{n-2}}}$

- Required to compute:
 - Treatment and control means
 - Treatment and control sample sizes
 - Treatment and control standard deviations (or some pooled SD)

Effect size calculation is not always trivial

Example: Cohen's d in a 2-armed RCT

• Bias correction (Hedges' g):
$$g = \frac{\Gamma(n^{-2}/2)}{\sqrt{\frac{n-2}{2}}\Gamma(n^{-3}/2)} \frac{\bar{Y}_T - \bar{Y}_C}{\sqrt{\frac{(n_T-1)s_T^2 + (n_C-1)s_C^2}{n-2}}}$$

• Approximate bias correction:
$$g \approx \frac{4n-12}{4n-9} \frac{\bar{Y}_T - \bar{Y}_C}{\sqrt{\frac{(n_T-1)s_T^2 + (n_C-1)s_C^2}{n-2}}}$$

• It is possible to obtain the pooled standard deviation via test statistics:

$$- t = \frac{\bar{Y}_T - \bar{Y}_C}{\sqrt{s^2 \left(\frac{1}{n_T} + \frac{1}{n_C}\right)}} \cong \frac{\bar{Y}_T - \bar{Y}_C}{\sqrt{\frac{(n_T - 1)s_T^2 + (n_C - 1)s_C^2}{n - 2}}} \frac{1}{\sqrt{\left(\frac{1}{n_T} + \frac{1}{n_C}\right)}}$$

• It is possible to get (approximate) pooled SD from the SE:

$$-SE \cong \sqrt{S^2 \left(\frac{1}{n_T} + \frac{1}{n_C}\right)}$$

Morthwestern Medicine® Feinberg School of Medicine

Effect size calculation is not always trivial

Example: (log) odds ratio in a 2-armed RCT

- $Y_T \sim B(n_T, \pi_T)$ and $Y_C \sim B(n_C, \pi_C)$ - $n_T + n_C = n$
- Odds ratio $\lambda = \frac{\pi_T/(1-\pi_T)}{\pi_C/(1-\pi_C)}$
- Estimate $\frac{Y_T/(n_T Y_T)}{Y_C/(n_C Y_C)}$ $z = log\left(\frac{Y_T/(n_T Y_T)}{Y_C/(n_C Y_C)}\right)$
- Asymptotically, $\log(z) \sim N(\log(\lambda), \frac{1}{Y_T} + \frac{1}{n_T Y_T} + \frac{1}{Y_C} + \frac{1}{n_C Y_C})$

A quick look at effect size calculations in R

Introduction to metafor

• Dagostino (1998) impact of antihistamines on runny nose severity for the common cold.

```
library(metafor)
dag es <- escalc(
  measure = "SMD", # "OR", "RR", etc.
  m1i = mt, # treatment means
  m<sub>2i</sub> = mc, # control means
  sd1i = sdt, # treatment SDs
  sd2i = sdc, # control SDs
  n1i = nt, # treatment group sample size
  n2i = nc # control group sample size
  data = metafor::dat.dagostino1998 %>%
filter(outcome == "rnic2")
```

What do we need to know about effect sizes?

- Effect parameters should be conceptually similar enough to consider jointly.
- Effect size indices need to be the same across studies.
 - What effect size index makes sense?
 - In our example, outcomes pertain to the same construct (runny nose), but are on different scales (e.g., 0-3, 0-4, 0-8, or 0-10).
 - We can put them on similar scales via Cohen's *d*.
 - It is often possible (with some normality assumptions) to convert from one scale to another:
 - *d* <--> log(odds ratio) <--> correlation
 - It may not be conceptually appropriate to change scales even if it is technically feasible.

How do we visualize data in a meta-analysis? Forest plot





How do we visualize data in a meta-analysis? Forest plot

quick meta-analysis fit, we'll come back to this later

```
remod <- rma(yi = yi, vi = vi, data = dag_es, method = "PM", knha = TRUE)
# make a forest plot
forest(remod, cex=.75, header="Study ID",</pre>
```

```
mlab="", slab = dag_es$study)
```



- We need effect estimates and variances (or SEs)
- They need to be on the same scale
 - We can often convert between effect size scales
- We should start by visualizing with a forest plot

M Northwestern Medicine[®]

Feinberg School of Medicine

Combining Effect Sizes

Goals of analyses, and basics for estimation

What is an effect size?

Some statistical considerations for multiple studies

- Estimands θ_i
 - "True" effects
 - Parameters
- Estimates T_i
 - Functions of the data
- Variances σ_i^2
 - Standard errors: σ_i
 - Sampling or estimation error variance that decrease with sample size
- Confidence/credible intervals

Assumptions about the studies/effects will govern if we do a *fixed-* or *randomeffects* meta-analysis



Fixed-effects meta-analysis

Strong assumptions ahead!

- Early statistical theory in the 1980s focused on fixed-effects models:
 - $\theta_1 = \theta_2 = \dots = \theta_k = \theta$
- Inferential goal: Estimate θ and report SE/CI, etc.



Morthwestern Medicine* Feinberg School of Medicine

Fixed-effects meta-analysis

Strong assumptions ahead!

- Early statistical theory in the 1980s focused on fixed-effects models:
 - $\theta_1 = \theta_2 = \dots = \theta_k = \theta$
- Assumes that studies are identical enough to produce identical effects.
 - Evidence from direct replications suggests we can't always do this even if we're explicitly trying to do so.



Morthwestern Medicine® Feinberg School of Medicine

Random-effects meta-analysis

Weaker assumptions

- Assumes $\theta_i \neq \theta_j$, instead the θ_i vary randomly:
 - $\theta_i \sim N(\mu, \tau^2)$
 - Need not be normal, but it's a common assumption.
- Assumes that studies are a random sample from some population.



Morthwestern Medicine[®] Feinberg School of Medicine

Random-effects meta-analysis

Weaker assumptions

- Assumes $\theta_i \neq \theta_j$, instead the θ_i vary randomly:
 - $\theta_i \sim N(\mu, \tau^2)$
 - Need not be normal, but it's a common assumption.
- Inferential goal: Estimate μ , τ^2 and report SE/CI, etc.
 - Report intervals likely to contain future values of θ_i (prediction interval)



Comparing fixed- and random-effects analyses

Fixed-effects analysis

- Estimate mean effect
 - Assumes common underlying effect across all studies
- One source of variation
 - Within-study (sampling) variation

Random-effects analysis

- Estimate mean effect
 - Mean of a distribution of effect parameters
 - Prediction interval for future effects
- Two sources of variation
 - Within-study (sampling) variation
 - Between-study variation
- Estimate between-study variation





Should I use fixed- or random-effects models?

• The Q-test for heterogeneity tests $H_0: \theta_1 = \theta_2 = ... = \theta_k$

$$- Q = \sum_{i=1}^{k} \frac{\left(T_{i} - \sum_{i=1}^{k} T_{i} / \sigma_{i}^{2} \right)^{2}}{\sigma_{i}^{2}} \sim \chi_{k-1}^{2}$$

- However, the Q-test has low power unless there are a large number of effects (k > 50-80).
- Unless there is a large # of effects, Q is not advised for discerning between model specification.
- Instead, choice should be consistent with beliefs about the studies
 - My default: random-effects
 - Caveat: there needs to be enough studies to estimate the between-study variation (k > 5-10)



- Target of inference is the distribution of the effect parameters characterized by μ and τ^2





- Target of inference is the distribution of the effect parameters characterized by μ and τ^2





- Target of inference is the distribution of the effect parameters characterized by μ and τ^2

Meta-analysis model

- $T_i \simeq N(\mu, \tau^2 + \sigma_i^2)$ $T_i = \mu + r_i + e_i$ where $r_i \simeq N(0, \tau^2)$ and $e_i \simeq N(0, \sigma_i^2)$
- UMVUE (and MLE) of $\boldsymbol{\mu}$

-
$$\overline{T}_{\cdot} = \frac{\sum_{i=1}^{k} w_i T_i}{\sum_{i=1}^{k} w_i}$$
 where $w_i = \frac{1}{\tau^2 + \sigma_i^2}$

- \overline{T} is asymptotically normal with variance $V[\overline{T}] = \frac{1}{\sum_{i=1}^{k} w_i}$
 - NHST $H_0: \mu = 0$
 - 95% CI for μ
- No UMVUE for τ^2
 - REML
 - Moment estimators: DerSimonian-Laird, Paule-Mandel, etc.

Meta-analysis model Estimation

- $T_i \sim N(\mu, \tau^2 + \sigma_i^2)$
- $\overline{T}_{.} = \frac{\sum_{i=1}^{k} w_i^* T_i}{\sum_{i=1}^{k} w_i^*}$ where $w_i^* = \frac{1}{\hat{\tau}^2 + \sigma_i^2}$

- \overline{T} is asymptotically normal with variance $\widehat{V}[\overline{T}] = \frac{1}{\sum_{i=1}^{k} w_i^*}$

- NHST $H_0: \mu = 0$
 - Use a Knapp-Hartung correction (like a t-test)
- 95% Cl for $\boldsymbol{\mu}$
- Use \overline{T} and $\hat{\tau}^2$ to make inferences about the *distribution* of future θ_i
 - 95% prediction interval

Weighting

Mean effects are estimated using $w_i^* = \frac{1}{\hat{\tau}^2 + \sigma_i^2}$

More weight goes to T_i with smaller σ_i^2

In a FE model, $\tau^2 = 0$, so $w_i^* = \frac{1}{\sigma_i^2}$

- Larger variation in weights
- Mean pulled harder toward some T_i

In a RE model, $w_i^* = \frac{1}{\hat{\tau}^2 + \sigma_i^2}$

- Less variation in weights than FE
- Mean pulled less strongly



Odds ratio (Fixed effect)



Odds ratio (Random effects)

A note on inference for between-study variance

- The scale of τ^2 depends on the scale of the θ_i
 - Alternatively, we can quantify τ^2 in a manner that is "scale-free" relative to the "typical" within-study variance σ^2
 - H^2 estimates $1 + \frac{\tau^2}{\sigma^2} = \frac{\tau^2 + \sigma^2}{\sigma^2} = \frac{total \, variation}{within-study \, varation}$
 - l^2 estimates $\frac{\tau^2}{\tau^2 + \sigma^2} = \frac{between study variation}{total variation}$
 - l^2 values > 30-40% are often considered "meaningful"
 - H² values > 1.33-1.75 are considered "large" or "meaningful"

A typical meta-anlysis

- Estimate of the mean effect $\boldsymbol{\mu}$
 - SE, Cl
 - NHST that $\mu = 0$
 - Use KNHA adjustment!
- Estimate of the variance
 - **-** l^2
 - $-H^{2}$
- Prediction Interval

remod <- rma(yi = yi, # effect estimates vi = vi, # variances of effect estimates data = dag_es, # dataset method = "PM", # use the Paule-Mandel RE model knha = TRUE # small-sample adjustment for tests) summary(remod) # view results predict(remod) # get prediction interval

A typical meta-anlysis Results

- Estimate of the mean effect μ Random-Effects Model (k = 9; tau^2 estimator: PM)
 - SE, CI
 - NHST that μ = 0
 - Use KNHA adjustment!
- Estimate of the variance
 - **-** l^2
 - $-H^{2}$
- Prediction Interval

logLik deviance AIC BIC AICc 1.1942 6.2132 1.6117 2.0061 3.6117

tau^2 (estimated amount of total heterogeneity): 0 (SE = 0.0181)
tau (square root of estimated tau^2 value): 0
I^2 (total heterogeneity / total variability): 0.00%
H^2 (total variability / sampling variability): 1.00

Test for Heterogeneity: Q(df = 8) = 6.2132, p-val = 0.6234

A typical meta-anlysis Results

- Estimate of the mean effect μ
 - SE, CI
 - NHST that $\mu = 0$
 - Use KNHA adjustment!
- Estimate of the variance
 - **-** l^2
 - $-H^{2}$
- Prediction Interval

estimate	se	tval	df	pval	ci.lb	ci.ub		
0.2539	0.0558	4.5480	8	0.0019	0.1252	0.3827	**	
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1								

Model Results:

A typical meta-anlysis Results

- Estimate of the mean effect $\boldsymbol{\mu}$
 - SE, CI
 - NHST that μ = 0
 - Use KNHA adjustment!
- Estimate of the variance
 - **-** l^2

$-H^2$	pred	se	ci.lb	ci.ub	pi.lb	pi.ub	
 Prediction Interval 	0.2539	0.0558	0.1252	0.3827	0.1252	0.3827	
Summarize everything with a forest plot

make a forest plot

A typical meta-analysis Results



M Northwestern Medicine[®] Feinberg School of Medicine

M Northwestern Medicine[®]

Feinberg School of Medicine

Selection and publication bias

Beware the published record

Are our effect sizes "representative"?

Rosenthal's File Drawer problem

 Systematic reviews are often dominated by published research.



- Are we only seeing a subset of relevant effect sizes?
 - Selective reporting within studies
 - "We reported the contrasts for which we found significant results"
 - Selective reporting of entire studies
 - "We didn't get a significant result so we didn't feel the need to publish"

Can we tell if we're missing "null" results? Funnel Plots

- Studies with high statistical power are unlikely to have null results (assuming effects are nonzero).
- Studies with low statistical power are more likely to have null results.
- Studies with low statistical power tend to have higher within-study variation.



Can we tell if we're missing "null" results? Funnel Plots

- Studies with high statistical power are unlikely to have null results (assuming effects are nonzero).
- Studies with low statistical power are more likely to have null results.
- Studies with low statistical power tend to have higher within-study variation.





funnel(remod)



Morthwestern Medicine[®] Feinberg School of Medicine

Tests for funnel plot asymmetry

- Egger's test
 - 1. Fit the model $T_i = \beta_0 + \beta_1 \sigma_i$
 - 2. Test $H_0: \beta_1 = 0$

Tests can also regress T_i on σ_i^2 or look at the rank correlation between $T_i \& \sigma_i$

regtest(remod) Regression Test for Funnel Plot Asymmetry

Model: mixed-effects meta-regression model Predictor: standard error

Test for Funnel Plot Asymmetry: t = 0.3521, df = 7, p = 0.7351 Limit Estimate (as sei -> 0): b = 0.2160 (CI: -0.0750, 0.5069)

Adjustments

- Trim and fill
- Selection models (likelihood based approach)
- Can be seen as outright corrections to biased parameter estimates, or as sensitivity analyses.
- Avoid p-curve, PEESE, and PET-PEESE

Trim and fill

- Trim: remove some of the oversampled significant results
 Fills incrusts ("residence" a consideration of the oversampled significant results
- Fill: impute "missing" nonsignficant results 🔶



Morthwestern Medicine* Feinberg School of Medicine

Selection models

- Likelihood based approach
 - Assume T_i are unconditionally normal
 - T_i is observed ($R_i = 1$) with some probability π given its *p*-value is <0.05 (3P model)
 - $p(T_i | R_i = 1) \propto$

$$\phi(T_i; \, \mu, \tau^2 + \sigma_i^2) \mathbf{1} \left\{ \frac{|T_i|}{\sigma_i} \ge 1.96 \right\} + \pi \phi(T_i; \, \mu, \tau^2 + \sigma_i^2) \mathbf{1} \left\{ \frac{|T_i|}{\sigma_i} < 1.96 \right\}$$

– Likelihood-based estimates for μ , τ^2 and π



M Northwestern Medicine[®] Feinberg School of Medicine



tf <- trimfill(remod)</th>Estimated number of missing studies on the left side: 1 (SE = 2.1192)summary(tf)redict(tf)tau^2 (estimated amount of total heterogeneity): 0 (SE = 0.0183)funnel(tf)tau (square root of estimated tau^2 value):0l^2 (total heterogeneity / total variability):0.00%

H² (total variability / sampling variability): 1.00

Test for Heterogeneity: Q(df = 9) = 7.8900, p-val = 0.5453

 Model Results:

 estimate
 se
 zval
 pval
 ci.lb
 ci.ub

 0.2386
 0.0622
 3.8338
 0.0001
 0.1166
 0.3606 ***

Example Selection model

weightr::weightfunct(
 estimate = dag_es\$yi,
 vi = dag_es\$vi,
 steps = c(0.05, 1)
}

Adjusted Model (k = 9):

tau^2 (estimated amount of total heterogeneity): 0.0000 (SE = 0.0195) tau (square root of estimated tau^2 value): 0.0000

Test for Heterogeneity: Q(df = 8) = 6.2132, p-val = 0.7184092

Model Results:

	estimate	std.error	z-stat	p-val	ci.lb	ci.ub
Intercept	0.2553	0.08518	2.9969	0.00273	0.08832	0.4222
0.05 < p < 1	1.0259	1.09665	0.9355	0.3495	-1.12346	3.1753

Likelihood Ratio Test: X^2(df = 1) = 0.0005748692, p-val = 0.98087

Summary of publication bias

- If it makes sense, conduct and report assessments of publication bias (funnel plots, Egger's or Begg's test)
- If there appears to be some publication bias, conduct and report adjustments
 - Trim-and-fill
 - Selection models (for larger k)
 - Avoid p-curve, PEESE, and PET-PEESE

Morthwestern Medicine

Feinberg School of Medicine

Meta-regression

It's just like regular regression...sort of

Summarizing conditional distributions

Are effects parameters related to observed covariates?

- Meta-regression concerns the relationship between effect sizes and observed covariates
 - How was a treatment implemented?
 - Where did the study take place?
 - On whom?
- Used to answer important questions:
 - What is the treatment effect in populations >65 years-old?
 - Does dosage matter for treatment effects?
 - Is the correlation stronger in some countries, but not others?
- Referred to sometimes as "subgroup analysis" or "meta-regression"

Meta-regression model

$$\theta_i = \mathbf{X}\beta + r_i \qquad T_i = \mathbf{X}\beta + r_i + e_i$$

- where $r_i \sim N(0, \tau^2)$ and $e_i \sim N(0, \sigma_i^2)$



Morthwestern Medicine[®] Feinberg School of Medicine

Estimation of meta-regression

- $T \sim N(\mathbf{X}\beta, \mathbf{W}^{-1})$ where $\mathbf{W} = \text{diag}(\tau^2 + \sigma_i^2)$
- Basic WLS estimate: $\hat{\beta} = (X'WX)^{-1}X'WT$
- In practice, use a moment-based estimator for $\tau^{\rm 2}$ and plug-in to estimate of β
 - REML, Paule-Mandel, DerSimonian-Laird

Weighted least squares for meta-regression



Feinberg School of Medicine

Inference for meta-regression

- Point estimates and SEs: $\hat{\beta}$ is consistent with variance $(X'WX)^{-1}$
- Omnibus test that all coefficients are 0
- Tests for individual coefficients
 - Knapp-Hartung corrections!
- Heterogeneity estimates (including *l*², *H*²)
- Prediction intervals (given X)

Example

Curtis 1998: Plant group and time of exposure

```
remod_mr <- rma(
    yi = yi,
    vi = vi,
    mods = ~ drug,
    data = dag_es,
    method = "PM",
    knha = TRUE
)
summary(remod_mr)
regtest(remod_mr)</pre>
```

```
weightr::weightfunc(
    estimate = dag_es$yi,
    vi = dag_es$vi,
    steps = c(0.05, 1)
```

Cannot do trim-and-fill for meta-regression

Example

Curtis 1998: Plant group and time of exposure

Mixed-Effects Model (k = 9; tau^2 estimator: PM)

tau^2 (estimated amount of residual heterogeneity): 0 (SE = 0.0193)
tau (square root of estimated tau^2 value): 0
I^2 (residual heterogeneity / unaccounted variability): 0.00%
H^2 (unaccounted variability / sampling variability): 1.00
R^2 (amount of heterogeneity accounted for): 0.00%

Test for Residual Heterogeneity: QE(df = 7) = 4.6875, p-val = 0.6980

Example

Curtis 1998: Plant group and time of exposure

Test of Moderators (coefficient 2): F(df1 = 1, df2 = 7) = 2.2783, p-val = 0.1749

Model Results:

	estimate	se	tval	df	pval	ci.lb	ci.ub	
intrcpt	0.3482	0.0812	4.2900	7	0.0036	0.1563	0.5401 *	*
drugdoxylamine	-0.1592	0.1055	-1.5094	7	0.1749	-0.4087	0.0902	

M Northwestern Medicine[®]

Feinberg School of Medicine

Summary

Some points to consider

- Most meta-analyses involve
 - Mean effect estimate + inference
 - Weighted averages
 - Knapp-Hartung corrections
 - Heterogeneity estimate + inference
 - $\tau^2, H^2, I^2,$
 - Checks of funnel plots/publication bias
 - Egger's test
 - Publication bias corrections
 - Trim-and-fill
 - Selection weighting
 - Meta-regression models

- Dependent effect sizes
 - Model within- and betweenstudy correlations
 - Robust variance estimation
- Missing data
 - FIML
 - Imputation

M Northwestern Medicine[®]

Feinberg School of Medicine

Thank you!



- <u>Metafor</u>
- Introduction to Meta-Analysis
- Handbook of Research Synthesis

